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On the Phase Diagram of the $SU(2)$ Adjoint Higgs Model in 2+1 Dimensions

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Abstract

The phase diagram is investigated for $SU(2)$ lattice gauge theory in $d = 3$, coupled to adjoint scalars. For small values of the quartic scalar coupling, λ , the transition separating Higgs and confinement phases is found to be first-order, in agreement with earlier work by Nadkarni. The surface of second-order transitions conjectured by Nadkarni, however, is shown instead to correspond to crossover behaviour. This conclusion is based on a finite size analysis of the scalar mass and susceptibility. The nature of the phase transition at the termination of first-order behaviour is investigated and we find evidence for a critical point at which the scalar mass vanishes. The photon mass and confining string tension are measured and are found to be negligibly small in the Higgs phase. This is correlated with the very small density of magnetic monopoles in the Higgs phase. The string tension and photon mass rise rapidly as the crossover is traversed towards the symmetric phase.

In this paper we report on our investigation of the phase diagram for $d = 3$ $SU(2)$ lattice gauge theory coupled to adjoint scalars. This theory is of interest for at least two reasons. Through dimensional reduction, it is related to $d = 4$ pure $SU(2)$ lattice gauge theory at high temperature. The model also contains in its Higgs phase the famous 't Hooft-Polyakov monopoles. The Higgs phase is continuously connected to the symmetric, confining phase. Thus the model may prove to be a useful laboratory for exploring confinement by monopoles. We will not further discuss monopole confinement here, but concentrate on the phase diagram and some general properties of the mass spectrum.

The continuum Euclidean action in the notation of Nadkarni [1] is given by

$$S = \int d^3x \left\{ \frac{1}{2g^2} \text{Tr}(F_{\mu\nu}F_{\mu\nu}) + \text{Tr}(D_\mu\phi D_\mu\phi) + m_0^2 \text{Tr}(\phi\phi) + \frac{\mu}{2} (\text{Tr}(\phi\phi))^2 \right\}, \quad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$, $D_\mu\phi = \partial_\mu\phi + i[A_\mu, \phi]$; and $F_{\mu\nu}$, A_μ , and ϕ are all traceless 2×2 Hermitian matrices ($\phi = \phi^a \sigma_a/2$, etc). The lattice form of the action is

$$\begin{aligned} S = & \beta \sum_{x, \mu > \nu} \left(1 - \frac{1}{2} \text{Tr} U_{\mu\nu}(x) \right) + 2 \sum_x \text{Tr}(\Phi(x)\Phi(x)) \\ & - 2\kappa \sum_{x, \mu} \text{Tr}(\Phi(x) U_\mu(x) \Phi(x + \hat{\mu}a) U^\dagger(x)) + \lambda \sum_x (2\text{Tr}(\Phi(x)\Phi(x)) - 1)^2 \end{aligned} \quad (2)$$

Here $U_\mu(x) = \exp(iagA_\mu(x))$ is the usual link variable, and $U_{\mu\nu}(x)$ is the corresponding plaquette. The continuum field $\phi(x)$ is related to the lattice field $\Phi(x)$ by $\phi(x) = \sqrt{\kappa/a} \Phi(x)$, where a is the lattice spacing, and the lattice parameters β and λ are defined by

$$\beta = \frac{4}{g^2 a}, \quad \lambda = \frac{\mu a \kappa^2}{8}. \quad (3)$$

The hopping parameter κ is determined by an equation relating it to the continuum mass parameter m_0^2 . The tree level form of this equation is

$$m_0^2 = \frac{2}{a^2} \left[\frac{1}{\kappa} - 3 - \frac{2\lambda}{\kappa} \right].$$

Dividing by a factor g^4 , and using Eq.(3), this can be rewritten as

$$\frac{m_0^2}{g^4} = \frac{\beta^2}{8} \left[\frac{1}{\kappa} - 3 - \frac{\kappa \mu}{\beta g^2} \right]. \quad (4)$$

Due to the superrenormalisability of the adjoint Higgs theory in $d = 3$, λ and g^2 do not require ultraviolet renormalisation. Suppose for the moment that this were true for m_0^2 as well. Eq.(4) then specifies how to adjust κ to correspond to given fixed ratios of the continuum parameters m_0^2/g^4 and μ/g^2 . The lattice spacing is set by $a = 4/g^2\beta$.

The mass parameter m_0^2 does of course require ultraviolet renormalisation, but only up to two loops. The known results in the \overline{MS} scheme [2] and in lattice perturbation theory [3] allow a generalisation of Eq.(4) to be derived,

$$\begin{aligned} \frac{m_0^2(g^2)}{g^4} &= \frac{\beta^2}{8} \left(\frac{1}{\kappa} - 3 - \frac{\kappa}{\beta} \frac{\mu}{g^2} \right) \\ &+ \frac{\Sigma\beta}{4\pi} \left(1 + \frac{5\mu}{8g^2} \right) \\ &+ \frac{1}{16\pi^2} \left[\frac{10\mu}{g^2} \left(1 - \frac{\mu}{4g^2} \right) \left(\ln \left(\frac{3\beta}{2} \right) + 0.09 \right) + 8.7 + \frac{5.8\mu}{g^2} \right], \end{aligned} \quad (5)$$

where $\Sigma = 3.17591$. The successive lines of this equation correspond to tree level, one loop and two loop terms, respectively. The dimensionful parameter in the \overline{MS} scheme has been chosen to be g^2 . Eq.(5) specifies how to approach the continuum limit ($\beta \rightarrow \infty, \lambda \rightarrow 0, \kappa \rightarrow 1/3$), with $m_0^2(g^2)/g^4$ and μ/g^2 held fixed. It becomes exact in the limit of vanishing lattice spacing.

Our analysis of the phase diagram is based on the early work of Nadkarni [1], who first investigated this model using a combination of numerical and analytic techniques. Following Nadkarni, we define the length ρ of the scalar field by $\rho = \sqrt{2\text{Tr}(\Phi\Phi)}$. Qualitatively the Higgs phase and symmetric phases can be distinguished by the value of $\langle\rho\rangle$, it being larger in the Higgs phase. The familiar language of symmetry breaking is useful in the Higgs phase, whereas the symmetric phase is better described as a confined hadronic phase. As was emphasized by Nadkarni, however, the two phases are in fact continuously connected, and there is really no fundamental distinction between them. When an actual phase transition separates the two phases, it is described by a surface $\kappa_c(\beta, \lambda)$, the Higgs (symmetric) phase lying above (below) the surface. The statement that the two phases are continuously

connected implies that this surface does not cover the entire $\beta - \lambda$ plane, so that any two points in the parameter space can be connected by a path which detours around it. While it is not feasible to map out the entire surface in a strictly numerical analysis, one of our principal results will be that the surface of phase transitions covers a much smaller region of the $\beta - \lambda$ plane than conjectured by Nadkarni. As discussed in more detail below, the vast region of the surface specified by Nadkarni as second-order corresponds rather to a crossover region which is without critical fluctuations or massless particles.

Our methods are similar to those used by two of the present authors in a previous study in $d = 3$ with $SU(2)$ gauge fields coupled to fundamental rather than adjoint scalars [4]. The link update used in [4] was modified by inserting a Metropolis step to take account of the quadratic dependence of the scalar term in Eq.(2) on the link variables. The Bunk [5] algorithm for the scalar update was used here also. With adjoint scalars, however, the approximate determination of Bunk's parameter α suggested in [5] and used in [4] occasionally resulted in a low acceptance rate for the scalar update (particularly at very low β). Solving the full cubic equation (Eq.(6) of [5]) for α gave acceptance rates of over 80% throughout the phase diagram.

The results presented here were obtained on lattices of size L^3 with $L = 8, 12, 16, 20, 24$ and 32 at $\beta = 6.0$ and 9.0 . A typical run consisted of $500 - 2000$ sweeps. The coupling $\beta = 6.0$ is already in the region of approximate scaling for $d = 3$ pure $SU(2)$ lattice gauge theory [6]. In the case with fundamental scalars [4], the mass spectrum at the nearby value of $\beta = 7.0$ has been calculated and has been found to scale very well to higher values of β . In the present case of adjoint scalars, we shall see later that there is good scaling of mass ratios between $\beta = 6.0$ and 9.0 . The procedure is, given κ and λ at say, $\beta = 6.0$, to use Eqs.(3) and (5) to determine the ratios of continuum parameters $m_0^2(g^2)/g^4$ and μ/g^2 . Then holding these ratios fixed, Eqs.(3) and (5) are solved again to find κ and λ at the new value of β , e.g. $\beta = 9.0$. Close enough to the continuum limit, physical mass ratios should be constant when moving along such a 'line of constant physics'.

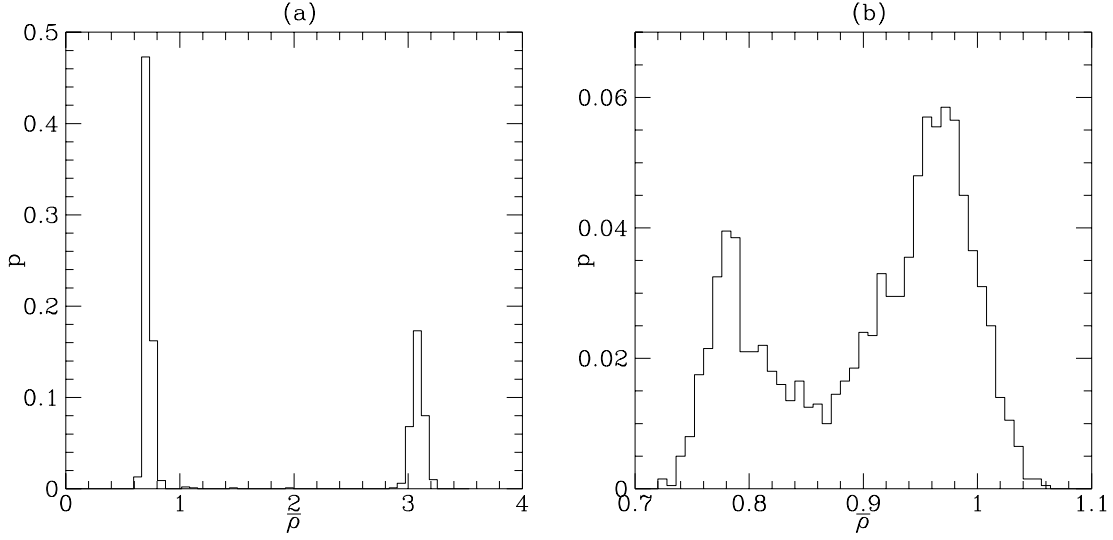


Figure 1: *Probability distributions of $\bar{\rho}$ indicating a first-order phase transition at $\beta = 6.0$. (a) $\kappa = 0.370473, \lambda = 0.001, L = 8$. (b) $\kappa = 0.38887, \lambda = 0.007, L = 20$.*

The most straightforward region of the phase diagram to analyse is where Higgs and symmetric phases are separated by a first-order transition. In agreement with Nadkarni, we find clear first-order behaviour for small λ , which for $\beta = 6.0$ means $\lambda \ll 0.01$. In Fig. 1(a) we show the histogram of $\bar{\rho}$ at $\beta = 6.0, \lambda = 0.001$ on an 8^3 lattice. $\bar{\rho}$ is the average over the lattice volume, V , of $\rho(x)$. Two peaks corresponding to symmetric and Higgs phases are clearly seen. Tunnelling between the two states occurred once in this run. When we increased the lattice size the peaks became so distinct that the to-and-fro tunnelling became unobservably rare. The first-order nature of the transition can also be seen from a plot of $\langle \bar{\rho} \rangle$ versus κ . There is a jump from smaller to larger values of $\langle \bar{\rho} \rangle$ which becomes rapidly sharper as the lattice volume is increased. The first-order behaviour observed at $\lambda = 0.001$ persists but weakens as λ increases. The two peaks in $\bar{\rho}$ become less widely separated and tunnelling between the two states becomes more frequent, *e.g.* Fig. 1(b) for $\lambda = 0.007$. Here the barrier is weak and we have frequent tunnelling, which persists even on fairly large

lattices. Clearly at this value of λ the first-order transition has become very weak and, as we shall see below, for slightly larger λ one loses all evidence for first-order behaviour. We see exactly the same behaviour in the corresponding calculations at $\beta = 9.0$.

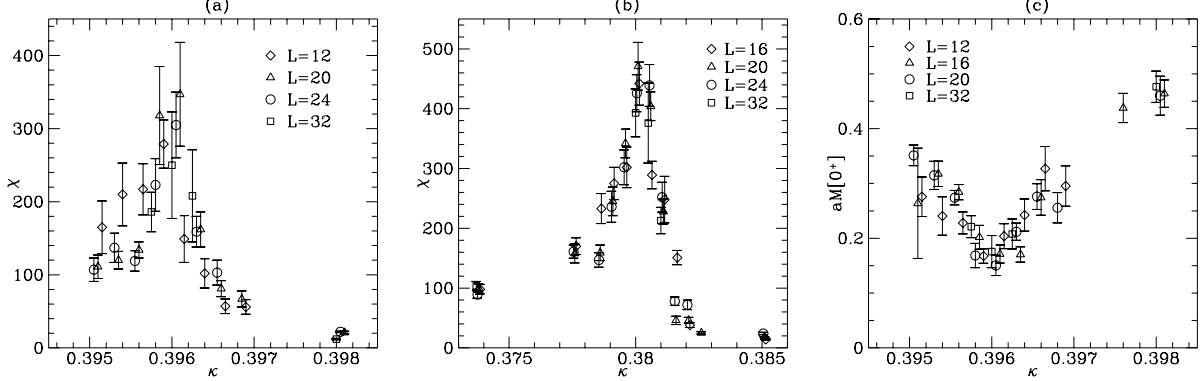


Figure 2: *Finite size scaling of scalar susceptibility and mass indicating crossover behaviour.* (a) $\beta = 6.0, \lambda = 0.01$. (b) $\beta = 9.0, \lambda = 0.01$. (c) $\beta = 6.0, \lambda = 0.01$.

Let us defer for the moment the question of what happens at the precise value of λ (for fixed β) at which first-order behaviour terminates. It was conjectured by Nadkarni that beyond the region of first-order behaviour, the surface $\kappa_c(\beta, \lambda)$ continues (all the way to $\lambda = \infty$ at $\beta = 6.0$ and 9.0), describing a second-order transition. We instead find no actual phase transition, only a smooth crossover behaviour. This conclusion is reached using a finite size analysis of the scalar mass and related susceptibility. Our scalar mass calculations are based on the operator $\rho_s(x) \equiv \rho^2(x)$ (and smeared versions thereof). The corresponding (normalised) susceptibility is

$$\chi = \frac{V}{a^3} \left(\frac{\langle \bar{\rho}_s^2 \rangle}{\langle \bar{\rho}_s \rangle^2} - 1 \right),$$

where V is the lattice volume. Fig. 2(a) shows how this susceptibility varies with κ at $\beta = 6.0$ and $\lambda = 0.01$. While there is a pronounced peak, it neither grows nor sharpens as V is increased (except perhaps for very small volumes). Fig. 2(b) shows a similar plot, with similar behaviour, taken at $\beta = 9.0$. In Fig. 2(c) we show how the lightest scalar mass

varies across this peak. (This is calculated from smeared versions of ρ_s ; the smearing is exactly as in [4] except that fundamental parallel transporters are replaced by their adjoint homologues.) According to a general finite size scaling analysis [7] a true second-order transition would be signalled by a peak in the susceptibility growing as $\chi_{max} \sim V^\gamma$ with some critical exponent γ . Likewise, across a second-order transition the lightest mass should go to zero with increasing lattice size, corresponding to a diverging correlation length. In the vicinity of the peak of the susceptibility we found the lightest scalar to be the lightest excitation. Although we do observe a susceptibility peak along with a minimum in the scalar mass, neither shows the required finite-size behaviour for a second-order transition. Instead we find $\chi, aM[0^+] \rightarrow \text{const}$ for $V \rightarrow \infty$, which is the expected behaviour for crossover. The same observation holds at still larger values of λ and at $\beta = 9.0$ as well.

Where the first-order transition ends and the crossover begins, one would expect to find critical behavior, *i.e.* diverging fluctuations and a vanishing mass for the lightest scalar. This would form a line of critical points in our three dimensional coupling constant space. To investigate this, we calculate the minimum value of the mass of the scalar as we traverse the crossover, and then move along the line of minima towards the first-order region. Suppose we work at fixed λ and β , and find the minimum scalar mass $m_{min}(\beta, \lambda)$ at $\kappa = \kappa_{min}(\beta, \lambda)$. If we now steadily decrease λ at fixed β , a critical point $\lambda_c(\beta)$, may occur at the boundary between crossover and first-order regions. This would be signalled by a vanishing of the scalar mass: $m_{min} \propto (\lambda - \lambda_c)^\nu$. We have performed such a study at $\beta = 6.0$ in which we have calculated m_{min} for three values of λ . Since m_{min} is small, it is crucial that we ensure that the lattices we use are large enough. Our scans through the crossover were carried out on a range of lattice sizes from 8^3 to 32^3 .

The results of these calculations are shown in Table 1. We observe that $m_{min}(\lambda)$ does decrease by about a factor of 2 when λ is reduced from 0.011 to 0.009. This is suggestive of a critical point at a value of $\lambda < 0.009$. Recall that we have performed calculations at $\beta = 6.0$ and $\lambda = 0.007$ which reveal a very weak first-order transition. This would place

| λ | κ_{min} | am_{min} |
|-----------|----------------|------------|
| 0.011 | 0.39820 (30) | 0.207 (23) |
| 0.010 | 0.39600 (25) | 0.161 (14) |
| 0.009 | 0.39380 (5) | 0.116 (8) |

Table 1: *Minimal scalar mass in the crossover region approaching λ_c at $\beta = 6.0$.*

the suspected critical point for $\beta = 6.0$ in the interval $0.007 < \lambda_c(6.0) < 0.009$. To locate it with more precision would require long runs on large lattices.

We consider now gauge bosons and magnetic monopoles. Where the scalar particles are in the fundamental representation of $SU(2)$, the existence of a residual or custodial $SU(2)$ global symmetry allows operators which couple directly to the triplet of gauge bosons to be constructed and the gauge boson masses determined from correlators of these operators [4]. For the present case of adjoint scalars, no such residual symmetry exists, and consequently there are no gauge-invariant operators which couple directly to charged gauge bosons. A gauge-invariant operator which directly couples to the photon in the Higgs phase can be constructed, however, starting with the continuum operator $2\text{Tr}(\phi F_{\mu\nu})$. In the Higgs phase, A_μ^3 is the electromagnetic gauge field in unitary gauge ($\phi = \phi^3 \sigma_3/2$). In this gauge our operator is $\sim F_{\mu\nu}^3$, and the latter is $\sim \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3$, since the gauge fields $A_\mu^{1,2}$ have only short range fluctuations. Thus in the Higgs phase, $2\text{Tr}(\phi F_{\mu\nu})$ couples to the field strength of the photon, and we can determine the photon mass by measuring its correlator [8]. Working as we do in momentum space, at least one of the momentum components p_μ, p_ν must be non-vanishing in order to retain that part of $2\text{Tr}(\phi F_{\mu\nu})$ which is $\sim \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3$.

On the lattice, we start from $\mathcal{F}_{\mu\nu} = 2\text{Tr}(\Phi(x)U_{\mu\nu}(x))$ which contains $2\text{Tr}(\phi F_{\mu\nu})$ in the continuum limit. The operator we use in practice has the field Φ inserted at all corners of the plaquette to maintain symmetry, and is then smeared to improve overlaps. We have measured the energy of the lowest state coupled to this operator at finite momentum.

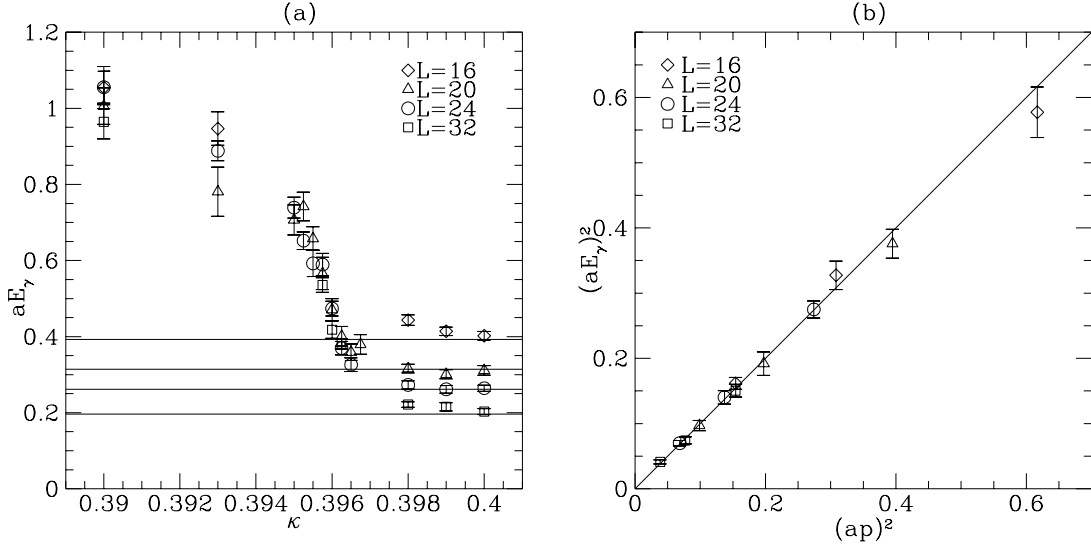


Figure 3: (a) Photon energy for $p = 2\pi/L$ at $\beta = 6.0, \lambda = 0.01$. The solid lines mark $p = 2\pi/L$ for the respective lattice sizes. (b) Dispersion relation for the photon at $\beta = 6.0, \lambda = 0.01, \kappa = 0.4$. The solid line marks the expectation for a massless photon.

Fig. 3(a) shows the energy, $E(p)$, for the lowest non-zero lattice momentum, $p_\mu = \frac{2\pi}{L}(1, 0)$ taken through the crossover at $\beta = 6.0, \lambda = 0.01$. As can be seen, once the value of κ is in the Higgs phase, *i.e.* $\kappa > \kappa_c(6.0, 0.01) \sim 0.396$, the energies agree very well with $2\pi/L$ which is the energy of a massless excitation on a lattice of side L . The results are equally good for all of the three lowest non-zero lattice momenta that we studied. To demonstrate this we plot in Fig. 3(b) $E^2(p)$ versus p^2 for all L and for the three lattice momenta at each L at a point well inside the Higgs phase. We see very striking evidence for the relativistic dispersion relation of a massless photon; a fit to the data yields $(aM_\gamma)^2 = 0.0010(20)$. In contrast, as we decrease κ and move into the symmetric phase, the energy rises rapidly as it should. In the symmetric phase the operator in question describes an object composed of adjoint scalars and gauge particles or ‘gluons’ which will have a mass on the hadronic scale. Equivalent results are obtained at $\beta = 9.0$ and other values of λ .

A photon of negligible mass in the Higgs phase is at first sight surprising. The classic work of Polyakov [8] established for adjoint scalars coupled to $SU(2)$ gauge fields, that the photon has a finite mass due to the presence of 't Hooft-Polyakov monopoles. This apparent paradox is resolved by noting that for the present range of parameters, monopoles have a very large action and the photon mass established by Polyakov is far too small to show up on the lattices studied here. An estimate for the action of a monopole is $S_m \sim 4\pi M_W/g^2$, where M_W is the mass of the charged vector boson in the Higgs phase [9]. We have not directly measured M_W as this would require gauge fixing (see the discussion above). We proceed by using the tree level expression $M_W^2/g^4 = -2m_0^2/(\mu g^2)$. Converting our parameter values $\beta = 6.0, \lambda = 0.01, \kappa \simeq 0.4$ to the continuum using Eq.(5) we find $S_m \approx 5.8$. Now Polyakov's formulas for the photon mass M_γ and monopole density n are [8],

$$M_\gamma^2 \sim \frac{2}{g^2} n, \quad n \sim \frac{(M_W)^{\frac{7}{2}}}{g} \exp(-S_m).$$

Evaluating these at $\beta = 6.0$ gives $(M_\gamma a)^2 \sim 10^{-4}$ and $na^3 \sim 10^{-4}$, so our small value for the photon mass is to be expected. We have measured the monopole density (in unitary gauge) using standard methods [10] in our configurations and find that there are indeed very few monopoles in the Higgs phase, as is displayed in Fig. 4(a). Of those that are there, all (within errors) are members of tightly bound dipoles, and therefore ineffective in generating a photon mass. The signal for dipoles is that the average flux from a monopole vanishes into noise within one or two lattice spacings away from the monopole location.

If, for all practical purposes, the Higgs phase does not contain any magnetic monopoles then as we cross over from the symmetric phase to the Higgs phase, the string tension (the coefficient of the linear piece of the confining potential) should vanish: $\sigma \rightarrow 0$. To investigate this we have calculated the mass of the lightest flux loop that joins onto itself through the periodic boundary of the lattice. If we have linear confinement then the mass, m_l , of such a flux loop should satisfy

$$am_l(L) = a^2 \sigma L - \frac{\pi}{6L}, \quad (6)$$

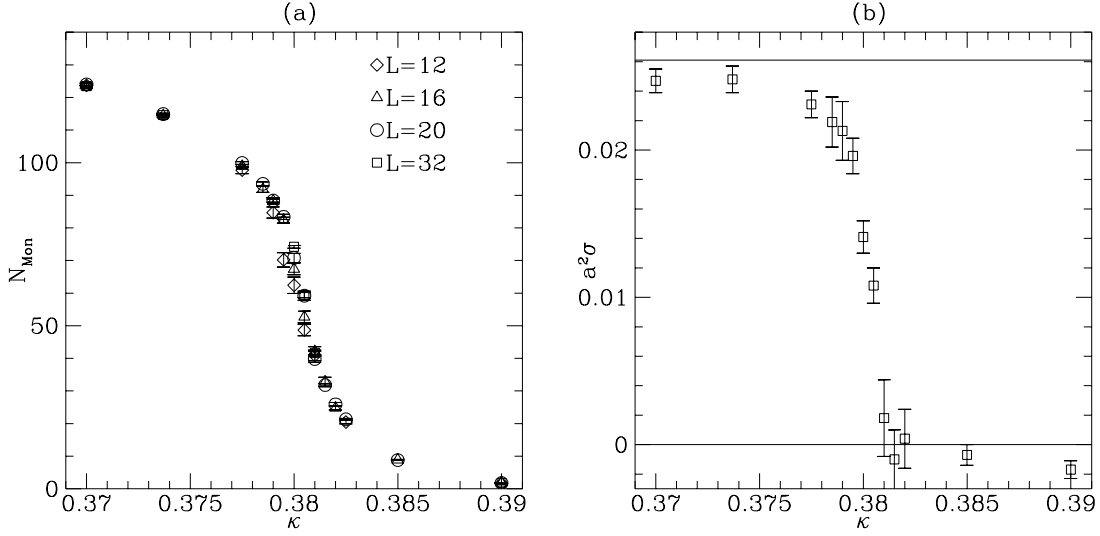


Figure 4: *Properties of the crossover region at $\beta = 9.0$, $\lambda = 0.01$. (a) Number of monopoles in a 12^3 cube. (b) String tension; the upper solid line marks that in the pure gauge theory.*

where we have included the well-known universal leading correction to the linear piece. We calculate $am_l(L)$ from the asymptotic exponential decay of correlations of smeared Polyakov loops on L^3 lattices exactly as in the pure gauge theory. In addition to the standard Polyakov loops, l_P , which we may schematically define by $l_P = \text{Tr}\{\prod_{t=1}^L U_t\}$, we also consider modified Polyakov loops obtained by inserting scalar fields; *i.e.* $l_P^s = \text{Tr}\{\Phi(L) \prod_{t=1}^L U_t\}$ and $l_P^{ss} = \text{Tr}\{\prod_{t=1}^L U_t \Phi(t)\}$. These operators have the same properties under the centre of the gauge group as the Polyakov loop and therefore project only onto states with non-trivial winding, such as our periodic flux loop. We mention this technical detail because, as we move through the crossover region from the symmetric to the Higgs phase, the best operators become l_P^s and then l_P^{ss} and one needs to use them rather than l_P in order to obtain accurate mass estimates. (Of course, this observation tells us something interesting about the dynamics of the crossover as well.)

We show in Fig. 4(b) a plot of our calculated value of the string tension, $a^2\sigma$, as we vary

κ through the crossover at $\beta = 9.0$ and $\lambda = 0.01$. The calculation was performed as follows. We calculated $m_l(L)$ for $L = 16, 20, 24$ and, for perhaps half the κ -values, for $L = 32$ as well. We then fitted these masses to the form Eq. (6). We obtained acceptable values of χ^2 for $\kappa \leq 0.3805$ and so for this range of κ this is how we obtained σ . (It is interesting to note that, throughout this range of κ , fits without the $O(1/L)$ string correction are very much poorer, even in the transition region where $a^2\sigma$ is rapidly decreasing. This provides strong statistical evidence for the presence of such a correction and, indeed, for its ‘universality’.) For $\kappa \geq 0.3810$, the value of χ^2 increased sharply. The confidence level of the best fit dropped from 90% at $\kappa = 0.3805$ to 0.5% at $\kappa = 0.3810$ and to $< 0.01\%$ at $\kappa \geq 0.3815$. In this range of κ we therefore allowed a constant piece and fitted the loop masses to $am_l(L) = c + a^2\sigma L$. Such fits had an acceptable χ^2 . (One can include a string correction as above, but it makes no significant difference.)

We see in Fig. 4(b) that the string tension collapses to zero around $\kappa \simeq 0.3805$. This is precisely the centre of the crossover (*i.e.* the coupling where the scalar mass attains its minimum value as we traverse the crossover at fixed β and λ). For $\kappa \geq 0.3810$, $m_l(L)$ is constant to a first approximation (apparently with a correction that seems to lead to a slight decrease with increasing L so that the fitted σ has a tendency to be slightly negative). This is in fact what we would expect in a Higgs phase with no monopoles. The flux loop consists of a unit flux traversing the $L \times L$ spatial plane in, say, the y -direction. In the Higgs phase we would expect there to be no flux ‘tube’ so that the flux simply spreads out uniformly in the x -direction. Moreover, since we are in the $U(1)$ -like Higgs phase, we neglect non-Abelian effects. Then the field strength is $E \sim \frac{1}{L}$, the energy density $\sim \frac{1}{L^2}$ and hence when we integrate this over the $L \times L$ plane we get a total energy that is independent of L . This matches what we observe and suggests that what happens to the confining flux tube as we traverse the centre of the crossover is that its width goes from being $O(\frac{1}{ag^2})$ to being $O(\frac{1}{L})$. This change takes place over a remarkably narrow range of couplings: between $\kappa = 0.379$ and $\kappa = 0.381$ in the case being considered here. Of course if we let $L \rightarrow \infty$

then there will be a very dilute gas of monopoles and this will reintroduce a finite, albeit extremely wide, flux tube and a non-zero, albeit extremely small, σ . Finally we remark that although the Higgs phase may have no visible string tension, it is still confining, albeit only logarithmically, because we are in two spatial dimensions. Thus $\langle l_P \rangle = 0$ for all values of the coupling and there is no *need* for a phase transition.

We return briefly to the lines of constant physics. At $\beta = 9.0$, $\kappa = 0.3737$, $\lambda = 0.01$ we are deep in the symmetric phase. Using a variety of operators we measure masses and the string tension as above. Now along a line of constant physics the ratio of a mass to the square root of the string tension should be independent of the lattice spacing scale, if the $O(a)$ corrections are small. To test this, we use Eqs.(3) and (5) to map $\kappa = 0.3737$, $\lambda = 0.01$ to the corresponding values at $\beta = 6.0$, and see whether the two sets of calculated mass ratios agree. This is indeed what we find for operators of purely gluonic, purely scalar or mixed origin (within errors which are $< 10\%$). Repeating this for a $\beta = 9.0$ point in the crossover region ($\lambda = 0.01, \kappa = 0.3805$), agreement is still good, although less so for the admittedly more noisy gluonic operators. Far inside the Higgs phase ($\lambda = 0.01, \kappa = 0.3900$) the string tension is vanishingly small. Mass ratios constructed using the lightest scalar mass (obtained from the purely scalar operator) as a reference do give good agreement for the operators of mixed origin, however. Thus we have good evidence for scaling between $\beta = 6.0$ and 9.0 , along lines of constant physics, and hence that our conclusions in this paper are relevant to the continuum theory.

In conclusion, we have clarified the phase diagram of the $SU(2)$ adjoint Higgs model in 2+1 dimensions. For small values of the scalar self-coupling, λ , the expected first-order transition indeed occurs. But as we increase λ this becomes a crossover rather than a second-order transition. At the point where the first-order transition vanishes there appears to be a critical point where the scalar mass vanishes. These calculations have been performed for values of the lattice spacing where deviations from the continuum limit are very small and so we believe that these are properties of the continuum theory. All this is,

of course, very much reminiscent of the behaviour of the theory with fundamental Higgs [12].

Understanding the phase diagram is an essential first step if we wish to use this theory as a testing ground for the monopole mechanism for confinement. The fact that the Higgs phase and the symmetric, confining phase are smoothly connected through a crossover region, makes us optimistic about the possibility of learning how confinement interpolates between the former phase, where it is well understood, [8], and the latter, where theoretical ideas exist, [11], but still remain speculative. This optimism is reinforced by the simple behaviour that we have found for the flux tube and the string tension as we pass through this crossover region. Of particular interest in this context is the critical point; it may be that we can find here a reasonably dense plasma of t'Hooft-Polyakov monopoles. These questions are under investigation.

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